

## Using an Ice Bath to Approximate the Triple Point of Water When Calibrating Secondary Standard Platinum Resistance Thermometers

Burns Engineering, Inc.  
10201 Bren Road East  
Minnetonka, MN 55311  
[www.burnsengineering.com](http://www.burnsengineering.com)  
[www.burnsengineering.com/BEnews](http://www.burnsengineering.com/BEnews)

### Abstract

The Resistance of a Secondary Standard Platinum Resistance Thermometer (SSPRT) at the Triple Point of Water (TPW) is the one of the most important performance criteria for proper monitoring and use of the thermometer. Using a TPW cell to make this measurement can present some challenges related to thermometer configuration and calibration efficiency. By approximating the TPW using a properly designed Ice Bath, greater flexibility in sensor configuration and improved throughput can be realized with minimal impact on the accuracy of the measurement. This paper discusses two methods for approximating the TPW resistance measurement using an ice bath, and a detailed uncertainty analysis to show what level of uncertainty can be achieved using each of these methods.

### 1. Introduction

There is no question that the Triple Point of Water, 0.01°C, is an important temperature point on the ITS-90 temperature scale. Those who are familiar with ITS-90 know that over the temperature range of 13.8033 K (-259.3467°C) to 1234.93 K (961.78°C), temperature is defined, in part, by the resistance ratio ( $W$ ) of a Standard Platinum Resistance Thermometer (SPRT) [1]. The resistance ratio is the resistance at temperature to the Resistance at the Triple Point of Water (RTPW).

$$\text{Ratio } (W) = (\text{Resistance at temperature}) / (\text{RTPW})$$

The RTPW is so important in monitoring the performance of a SPRT that it is common practice to take multiple RTPW measurements over the course of a calibration to insure the SPRT is working properly. Subsequent RTPW measurements taken on the SPRT at regular intervals during its in-service period can add assurance that the SPRT is continuing to work properly, or has developed a problem and needs to be removed from service. The same is true for Secondary Standard PRTs and Industrial PRTs used as calibration standards. Tracking the resistance at the triple point of water through periodic measurements on these units is highly recommended as it can greatly reduce the risk of using problematic PRTs as reference standards.

Secondary Standard Platinum Resistance Thermometers (SSPRTs) and Industrial Platinum Resistance Thermometers (IPRTs) are commonly used by laboratories as working standards when the uncertainty budget permits. Even though these thermometers do not perform at the level of a SPRT, they have a big advantage in that they are more rugged and less expensive than SPRTs. The increased ruggedness comes at the expense of accuracy. Given the accuracy of these

thermometers, there is more flexibility in the methods used to calibrate them. While it may be possible to calibrate these thermometers in a TPW cell, the user may not need the low uncertainty or want to deal with the increased expense and complication associated with this method. There may also be sensor configuration complications where the geometry of the sensor makes using a traditional TPW cell impossible.

Two alternate methods have been used to approximate the RTPW value on SSPRTs and IPRTs. Both methods involve the use of an ice bath. One method is to perform a direct comparison of the Unit Under Test (UUT) to a SPRT in an ice bath. The other method is to determine the resistance of the UUT using an ice bath as a 0°C reference source and adding a nominal ohmic correction factor to account for the 0.01°C temperature difference between the nominal ice bath source temperature and the TPW. Either of these methods could be considered appropriate as long as an acceptable test uncertainty ratio (TUR), typically 4:1, can be achieved.

## 2. Test Units

Testing was performed on a total of 10 thermometers as described below. The accuracy value listed is the thermometer accuracy at the TPW.

- One metal sheath 25.5 ohm SPRT with an accuracy of 1 mK.
- One quartz sheath 25.5 ohm SPRT with an accuracy of 1 mK.
- Four 100 ohm Secondary Standard PRTs with a nominal alpha coefficient of .003925 ohm/ohm/°C and an accuracy of 18 mK.
- Four 100 ohm industrial PRTs with a nominal alpha coefficient of .003851 ohm/ohm/°C and an accuracy of 50 mK.

These PRTs were chosen for this test because they are commercially available and are regularly used by laboratories as reference thermometers. Note that the accuracies listed take into account the short term performance of the thermometer when used over its rated temperature range.

## 3. Methods

The following methods were used to determine the RTPW values for these thermometers. Three measurements were made on each thermometer using each method, resulting in a total of nine RTPW values for each thermometer.

### Method 1: Triple Point of Water Method

For this method, a triple point of water cell was used as the temperature source. The test PRT was inserted into the TPW cells' thermometer well and allowed to stabilize prior to making the measurements. The resistance ratio of the UUT to a standard resistor was determined using a precision AC thermometry bridge with 1 ppm accuracy. The resistance ratio was measured using a 1 mA sensing current and a 100 ohm standard resistor which is maintained at  $25.0 \pm 0.1$  °C in an oil bath. The measured ratio was then multiplied by the resistance value of the standard resistor to determine the resistance of the UUT. This method is capable of the lowest uncertainty of the three methods used but requires the preparation and use of a triple point of water cell which can be inconvenient and can restrict the size of the PRTs which may be tested. Detailed information on the proper use of a water triple point cell can be found in ASTM Standard E1750 – *Standard Guide for Use of Water Triple Point Cells* [2].

### Method 2: Comparison Calibration Method

For this method, an ice bath prepared using distilled water and ice made from distilled water, was used as the temperature source. The PRT to be calibrated along with a 25.5 ohm SPRT were immersed into the ice bath and allowed to stabilize. The resistance ratio of the UUT to the SPRT was determined using the same precision AC thermometry bridge listed in Method 1. The resistance ratio was measured using a 1mA sensing current. This ratio was then multiplied by the resistance value of the SPRT at the TPW to determine the resistance of the UUT at the TPW. This method results in a larger uncertainty than is possible using a TPW cell primarily due to the stability and uniformity of the ice bath. In some instances this method could still be considered undesirable as it requires the use of a calibrated SPRT which is a very delicate instrument. The advantages of this method, however, are that it is adaptable to varying thermometer configurations, and it does not require the use of a standard resistor or maintenance bath. An additional advantage is that because of the use of the SPRT, the results are relatively immune to the actual temperature of the ice bath and therefore the purity of the ice and water are considered to have an insignificant affect on the uncertainty.

### Method 3: Ice Bath as 0°C Source Method

For this method, an ice bath prepared using distilled water and ice made from distilled water, was used as a 0°C fixed temperature source. The PRT to be calibrated was immersed into the ice bath and allowed to stabilize. The resistance ratio of the UUT to a standard resistor was determined using the same AC thermometry bridge listed in Method 1. The resistance ratio was measured using a 1 mA sensing current and a 100 ohm standard resistor which is maintained at  $25.0 \pm 0.1^\circ\text{C}$  in an oil bath. The measured ratio is then multiplied by the resistance value of the standard resistor to determine the resistance of the UUT at 0°C. To obtain the resistance at 0.01°C a nominal ohmic correction as shown in Table 1 was added to the measured 0°C resistance.

Table 1. Nominal ohmic correction for ice bath method.

<b>UUT</b>	<b>Nominal RTPW</b>	<b>Ohmic Correction</b>
SPRT	25.5 ohms	0.0010
SSPRT	100 ohms	0.0040
IPRT	100 ohms	0.0039

This method could be considered the most desirable because it does not require the use of a TPW cell or a delicate SPRT, and is adaptable to many different physical configurations of PRTs. However, the purity of the water and ice used to prepare the bath has a significant impact on the uncertainty attainable using this method, more information on the proper preparation and use of an ice bath as a reference temperature can be found in ASTM Standard E563 – *Standard Practice for Preparation and Use of an Ice-Point Bath as a Reference Temperature* [3]. This method could be further simplified by using a digital ohmmeter to directly measure the resistance of the UUT, however the accuracy of the meter must be selected to meet the user’s uncertainty requirements.

## **4. Uncertainty Estimate**

An uncertainty estimate was performed for each of the three different types of PRTs in each of the three different methods. This was done because the performance of the UUT must be taken

into account, and each of the three types of PRTs has a different performance capability. The uncertainty sources that were identified and included in the estimate are:

- Triple point of water cell uncertainty
- Triple point of water cell reproducibility
- Reference SPRT uncertainty
- Reference SPRT drift
- Bath stability and uniformity
- Bridge uncertainty
- Bridge resolution
- Standard resistor uncertainty
- Standard resistor drift
- Standard resistor thermal effects
- Mathematical ohmic correction error
- Repeatability and reproducibility

Table 2 summarizes the results of the detailed analyses and is followed by some notes of interest regarding the uncertainty estimation. The detailed analyses can be found in Appendix A.

Table 2. Summary of uncertainties of different methods.

<b>PRT Type</b>	<b>Method 1 TPW Cell</b>	<b>Method 2 Comparison Cal</b>	<b>Method 3 Ice Bath as 0°C Source</b>
SPRT	1.6 mK	3.7 mK	3.5 mK
SSPRT	1.2 mK	3.5 mK	3.3 mK
IPRT	1.9 mK	3.5 mK	3.3 mK

Notes on Uncertainty Estimates.

Examining the uncertainty between methods shows method 1 had the lowest uncertainty, as expected. The uncertainties for methods 2 and 3 were larger than method 1 by up to 2.3 mK, but were all within a few tenths of a mK of each other. Examining the uncertainty between PRT types within a method shows less variability than between methods. What was somewhat unexpected was that the SPRT did not have the lowest uncertainty. One of the reasons for this has to do with the uncertainty and resolution of the bridge when measuring the SPRT ratio, which is lower than the ratio with the other two types of PRTs. The lower ratio can result in a larger contribution to the uncertainty, particularly when the ratio is less than 1. See the tables in Appendix A for details.

In a typical uncertainty analysis it would be expected that, given the same equipment and method, the most significant difference in the uncertainties between thermometers would come from the performance of the UUT, specifically the short term repeatability and hysteresis. For a SPRT the short term repeatability and hysteresis is better than it is for a SSPRT or an IPRT. The lower the short term repeatability and hysteresis of the UUT, the lower the contribution to uncertainty from the Repeatability and Reproducibility (R&R) component. For this testing, all three types of UUTs exhibited a low R&R component primarily because the UUTs were not exposed to any temperature extremes in between the measurements. If these same units were to be calibrated over their full rated temperature range, the SSPRT and IPRT would have much larger uncertainties than the SPRT because the R&R component would be larger due to the

UUTs actual repeatability and hysteresis. This would be true regardless of which method was used to measure the RTPW value.

## 5. Results

To compare the results between the three different methods, a baseline value needed to be established for each PRT. This was done by determining the average value of the three measurements made using the TPW cell, Method 1. This value was chosen because the uncertainty of Method 1 was the lowest of the three methods. The difference between the average from Method 1 and each individual measurement was determined in ohms and then that value was converted to an equivalent mK difference by using each thermometer's nominal sensitivity at 0°C. Figure 1 below shows the difference in the result by thermometer type and method.

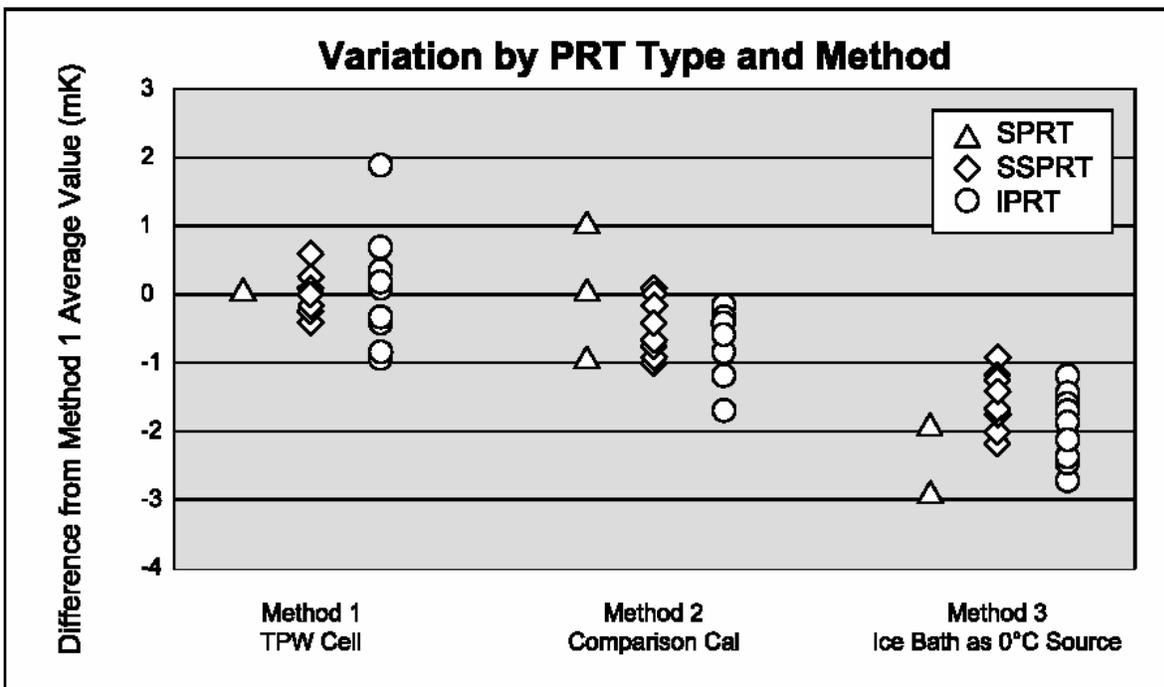


Figure 1. Variation in TPW measurements by type and method.

It is clear from this figure that both methods 2 and 3 yield results that are relatively close to the results obtained using the TPW cell. The following observations are worth noting.

Method 1 showed no variation in the values obtained for either of the SPRT units tested. The likely cause is that both of these units perform so well that the equipment used is not sensitive enough to measure the slight variations caused by these thermometers and the TPW cell. The data for the SSPRT and IPRT using Method 1 does exhibit some spread which is likely caused by the short term repeatability of these units.

Method 2 shows variability in the readings of all three types of thermometers but they all fall within a  $\pm 2$  mK grouping from the average value obtained using the TPW cell.

Method 3 shows a comparable variability to Method 2 except the mean values are biased low by approximately 1.5 to 2.5 mK. The bias is likely caused by the purity of the ice and water used to prepare the bath resulting in a bath temperature that is actually low by a few mK. All of the readings using this method fall within  $\pm 3$  mK from the average value obtained using the TPW cell.

## **6. Conclusion**

These results indicate that any of the three methods could be used to verify proper performance of the SSPRT or IPRT. For the SSPRT, which has an accuracy of 18 mK at the TPW, it would be desirable to use a method with an uncertainty of better than 4.5mK to achieve a TUR of 4:1. For the IPRT, which has an accuracy of 50 mK at the TPW, it would be desirable to use a method with an uncertainty better than 12.5mK to achieve a 4:1 TUR. All three of the methods used have adequate uncertainty to support these TURs. Verification of the SPRT performance is best accomplished using the TPW cell unless the full accuracy of the thermometer is not required.

## **7. Summary**

Periodic measurements of the resistance at the triple point of water on any PRT that is used as a calibration standard is highly recommended to insure the PRT is working properly between scheduled calibrations. The appropriate method for making this measurement should be selected based on the accuracy of the thermometer and the required uncertainty of the measurement. The traditional method of using a TPW cell, while sufficiently accurate, can be inconvenient, inefficient, and restrictive. Two alternative methods which use an ice bath as a temperature source have been described, along with detailed uncertainty analyses and test data, that demonstrates the capability of these methods to be better than  $\pm 4$  mK at the TPW. Using these alternate methods can be more convenient, efficient, and adaptable than using a TPW cell.

## **Appendix A**

### **Detailed Uncertainty Analysis**

#### **Appendix Overview**

The following tables give the details of the uncertainty analysis for each thermometer type using each method.

- Method 1 which uses the TPW cell,
  - o See tables A.1A, A.1B, and A.1C.
- Method 2 which uses a comparison calibration against an SPRT,
  - o See tables A.2A, A.2B, and A.2C.
- Method 3 which uses an ice bath as a 0°C reference temperature,
  - o See tables A.3A, A.3B, and A.3C.

Table A.1A. Uncertainty analysis of a SPRT in a TPW cell (Method 1).

Source	Source Uncertainty	Type (A/B)	Level of Confidence (%)	k factor	Standard Uncertainty (mK)
<b>TPW Cell Uncertainty</b> Manufacturer certification is 0.1 mK maximum. This is assumed as a 95% (k=2) coverage.	0.1 mK	B	95.45	2	0.05
<b>TPW Cell Reproducibility</b> Manufacturer certification is 0.02 mK maximum. This is assumed as a 95% (k=2) coverage.	0.02 mK	B	95.45	2	0.01
<b>Bridge Uncertainty</b> Bridge spec is 1ppm or 1 digit. For this measurement 1 digit is larger, this equates to 1.0 mK. This is assumed as a 95% (k=2) coverage.	1.0 mK	B	95.45	2	0.50
<b>Bridge Resolution</b> Bridge spec is 1 ppm, the uncertainty limit is ½ of this which equates to 0.5 mK. This is assumed as a 100% confidence rectangular distribution, (k=1.732)	0.5 mK	B	100	1.732	0.29
<b>Standard Resistor Uncertainty</b> Calibration certificate for resistor states an expanded uncertainty of less than 4 ppm with a 95% (k=2) coverage. This equates to 1.0 mK.	1.0 mK	B	95.45	2	0.50
<b>Standard Resistor Drift</b> The drift of the standard resistor has been tracked for multiple years and has performed at better than 1 ppm per year drift, this is assumed as a 95% (k=2) coverage. This equates to 0.25 mK.	0.25 mK	B	95.45	2	0.13
<b>Standard Resistor Thermal Effects</b> The resistor is maintained in a temperature controlled bath that results in 0.3 ppm variation in the resistance, this equates to 0.08 mK. This is assumed as a "U-Shaped" distribution with 100% coverage, k=1.414	0.08 mK	B	100	1.414	0.06
<b>Combined Standard Uncertainty for Equipment Only</b> This does not include Repeatability and Reproducibility component which can be heavily influenced by the UUT.					0.78
<b>Repeatability and Reproducibility</b> R&R for this UUT using this method is 0.00 mK. The most likely reason for this is that the equipment used is not capable of discerning the small differences in the method.	0.00 mK	A	63.2	1	0.00
<b>Standard Uncertainty = RSS Total =</b>					<b>0.78</b>
<b>Expanded Uncertainty, 95% (k=2)</b>					<b>1.6 mK</b>

Table A.1B. Uncertainty analysis of a SSPRT in a TPW cell (Method 1).

Source	Source Uncertainty	Type (A/B)	Level of Confidence (%)	k factor	Standard Uncertainty (mK)
TPW Cell Uncertainty Manufacturer certification is 0.1 mK maximum. This is assumed as a 95% (k=2) coverage.	0.1 mK	B	95.45	2	0.05
TPW Cell Reproducibility Manufacturer certification is 0.02 mK maximum. This is assumed as a 95% (k=2) coverage.	0.02 mK	B	95.45	2	0.01
Bridge Uncertainty Bridge spec is 1ppm or 1 digit. For this measurement 1 ppm is the same as 1 digit, this equates to 0.25 mK. This is assumed as a 95% (k=2) coverage.	0.25 mK	B	95.45	2	0.13
Bridge Resolution Bridge spec is 1 ppm, the uncertainty limit is ½ of this which equates to 0.13 mK. This is assumed as a 100% confidence rectangular distribution, (k=1.732).	0.13 mK	B	100	1.732	0.07
Standard Resistor Uncertainty Calibration certificate for resistor states an expanded uncertainty of less than 4 ppm with a 95% (k=2) coverage. This equates to 1.0 mK.	1.0 mK	B	95.45	2	0.50
Standard Resistor Drift The drift of the standard resistor has been tracked for multiple years and has performed at better than 1 ppm per year drift, this is assumed as a 95% (k=2) coverage. This equates to 0.25 mK.	0.25 mK	B	95.45	2	0.13
Standard Resistor Thermal Effects The resistor is maintained in a temperature controlled bath that results in 0.3 ppm variation in the resistance, this equates to 0.08 mK. This is assumed as a “U-Shaped” distribution with 100% coverage, k=1.414	0.08 mK	B	100	1.414	0.06
<b>Combined Standard Uncertainty for Equipment Only</b> This does not include Repeatability and Reproducibility component which can be heavily influenced by the UUT.					0.54
Repeatability and Reproducibility R&R for this UUT using this method is 0.26 mK.	0.26 mK	B	63.2	1	0.26
Standard Uncertainty = RSS Total =					0.60
Expanded Uncertainty, 95% (k=2)					1.2 mK

Table A.1C. Uncertainty analysis of an IPRT in a TPW cell (Method 1).

Source	Source Uncertainty	Type (A/B)	Level of Confidence (%)	k factor	Standard Uncertainty (mK)
<b>TPW Cell Uncertainty</b> Manufacturer certification is 0.1 mK maximum. This is assumed as a 95% (k=2) coverage.	0.1 mK	B	95.45	2	0.05
<b>TPW Cell Reproducibility</b> Manufacturer certification is 0.02 mK maximum. This is assumed as a 95% (k=2) coverage.	0.02 mK	B	95.45	2	0.01
<b>Bridge Uncertainty</b> Bridge spec is 1ppm or 1 digit. For this measurement 1 ppm is the same as 1 digit, this equates to 0.25 mK. This is assumed as a 95% (k=2) coverage.	0.25 mK	B	95.45	2	0.13
<b>Bridge Resolution</b> Bridge spec is 1 ppm, the uncertainty limit is ½ of this which equates to 0.13 mK. This is assumed as a 100% confidence rectangular distribution, (k=1.732).	0.13 mK	B	100	1.732	0.07
<b>Standard Resistor Uncertainty</b> Calibration certificate for resistor states an expanded uncertainty of less than 4 ppm with a 95% (k=2) coverage. This equates to 1.0 mK.	1.0 mK	B	95.45	2	0.50
<b>Standard Resistor Drift</b> The drift of the standard resistor has been tracked for multiple years and has performed at better than 1 ppm per year drift, this is assumed as a 95% (k=2) coverage. This equates to 0.25 mK.	0.25 mK	B	95.45	2	0.13
<b>Standard Resistor Thermal Effects</b> The resistor is maintained in a temperature controlled bath that results in 0.3 ppm variation in the resistance, this equates to 0.08 mK. This is assumed as a “U-Shaped” distribution with 100% coverage, k=1.414	0.08 mK	B	100	1.414	0.06
<b>Combined Standard Uncertainty for Equipment Only</b> This does not include Repeatability and Reproducibility component which can be heavily influenced by the UUT.					0.54
<b>Repeatability and Reproducibility</b> R&R for this UUT using this method is 0.80 mK.	0.80 mK	A	63.2	1	0.80
<b>Standard Uncertainty = RSS Total =</b>					<b>0.97</b>
<b>Expanded Uncertainty, 95% (k=2)</b>					<b>1.9 mK</b>

Table A.2A. Uncertainty analysis of a SPRT by comparison to a SPRT (Method 2).

Source	Source Uncertainty	Type (A/B)	Level of Confidence (%)	k factor	Standard Uncertainty (mK)
<b>Uncertainty of reference SPRT Resistance</b> The standard uncertainty of the RTPW value used for this measurement is 0.000078 ohms, this equates to .78 mK. This includes short term drift.	0.78 mK	B	63.1	1	0.78
<b>Drift of reference SPRT Resistance</b> Short term drift is included in value above. Long term drift is eliminated since RTPW is updated daily.	0.00 mK	B	95.45	2	0.00
<b>Bridge Uncertainty</b> Bridge spec is 1 ppm or 1 digit, which ever is larger. For this measurement 1 ppm is the same as 1 digit. 1 ppm equates to 0.25 mK This is assumed as a 95% (k=2) coverage.	0.25 mK	B	95.45	2	.13
<b>Bridge Resolution</b> Bridge spec is 1 ppm, the uncertainty limit is ½ of this which equates to 0.13 mK. This is assumed as a 100% confidence rectangular distribution, (k=1.732).	0.13 mK	B	100	1.732	0.07
<b>Ice Bath Stability and Uniformity</b> Periodic testing of our ice bath shows that it is consistently within 3.0 mK of 0°C. For this analysis stability and uniformity will be considered to be 3.0 mK with a 95% (k=2) coverage.	3.0 mK	B	95.45	2	1.50
<b>Combined Standard Uncertainty for Equipment Only</b> This does not include Repeatability and Reproducibility component which can be heavily influenced by the UUT.					1.70
<b>Repeatability and Reproducibility</b> R&R for this UUT using this method is 0.74 mK.	0.74 mK	A	63.2	1	0.74
<b>Standard Uncertainty = RSS Total =</b>					1.85
<b>Expanded Uncertainty, 95% (k=2)</b>					3.7 mK

Table A.2B. Uncertainty analysis of a SSPRT by comparison to a SPRT (Method 2).

Source	Source Uncertainty	Type (A/B)	Level of Confidence (%)	k factor	Standard Uncertainty (mK)
<b>Uncertainty of reference SPRT Resistance</b> The standard uncertainty of the RTPW value used for this measurement is 0.000078 ohms, this equates to 0.78 mK. This includes short term drift.	0.78 mK	B	63.1	1	0.78
<b>Drift of reference SPRT Resistance</b> Short term drift is included in value above. Long term drift is eliminated since RTPW is updated daily.	0.00 mK	B	95.45	2	0.00
<b>Bridge Uncertainty</b> Bridge spec is 1 ppm or 1 digit, which ever is larger. For this measurement 1 ppm is larger, 1 ppm equates to 0.25 mK This is assumed as a 95% (k=2) coverage.	0.25 mK	B	95.45	2	.13
<b>Bridge Resolution</b> Bridge spec is 1 ppm, the uncertainty limit is ½ of this which equates to 0.13 mK. This is assumed as a 100% confidence rectangular distribution, (k=1.732).	0.13 mK	B	100	1.732	0.07
<b>Ice Bath Stability and Uniformity</b> Periodic testing of our ice bath shows that it is consistently within 3.0 mK of 0°C. For this analysis stability and uniformity will be considered to be 3.0 mK with a 95% (k=2) coverage.	3.0 mK	B	95.45	2	1.50
<b>Combined Standard Uncertainty for Equipment Only</b> This does not include Repeatability and Reproducibility component which can be heavily influenced by the UUT.					1.70
<b>Repeatability and Reproducibility</b> R&R for this UUT using this method is 0.36 mK	0.36 mK	A	63.2	1	0.36
Standard Uncertainty = RSS Total =					1.73
Expanded Uncertainty, 95% (k=2)					3.5 mK

Table A.2C. Uncertainty analysis of an IPRT by comparison to a SPRT (Method 2).

Source	Source Uncertainty	Type (A/B)	Level of Confidence (%)	k factor	Standard Uncertainty (mK)
<b>Uncertainty of reference SPRT Resistance</b> The standard uncertainty of the RTPW value used for this measurement is 0.000078 ohms, this equates to 0.78 mK. This includes short term drift.	0.78 mK	B	63.1	1	0.78
<b>Drift of reference SPRT Resistance</b> Short term drift is included in value above. Long term drift is eliminated since RTPW is updated daily.	0.00 mK	B	95.45	2	0.00
<b>Bridge Uncertainty</b> Bridge spec is 1 ppm or 1 digit, which ever is larger. For this measurement 1 ppm is larger, 1 ppm equates to 0.25 mK This is assumed as a 95% (k=2) coverage.	0.25 mK	B	95.45	2	.13
<b>Bridge Resolution</b> Bridge spec is 1 ppm, the uncertainty limit is ½ of this which equates to 0.13 mK. This is assumed as a 100% confidence rectangular distribution, (k=1.732).	0.13 mK	B	100	1.732	0.07
<b>Ice Bath Stability and Uniformity</b> Periodic testing of our ice bath shows that it is consistently within 3.0 mK of 0°C. For this analysis stability and uniformity will be considered to be 3.0 mK with a 95% (k=2) coverage.	3.0 mK	B	95.45	2	1.50
<b>Combined Standard Uncertainty for Equipment Only</b> This does not include Repeatability and Reproducibility component which can be heavily influenced by the UUT.					1.70
<b>Repeatability and Reproducibility</b> R&R for this UUT using this method is 0.51 mK.	0.51 mK	A	63.2	1	0.51
Standard Uncertainty = RSS Total =					1.77
Expanded Uncertainty, 95% (k=2)					3.5 mK

Table A.3A. Uncertainty analysis of a SPRT by ice bath as 0°C reference source (Method 3).

Source	Source Uncertainty	Type (A/B)	Level of Confidence (%)	k factor	Standard Uncertainty (mK)
<b>Ice Bath Stability and Uniformity</b> Periodic testing of our ice bath shows that it is consistently within 3.0 mK of 0°C. For this analysis stability and uniformity will be considered to be 3.0 mK with a 95% (k=2) coverage.	3.0 mK	B	95.45	2	1.50
<b>Bridge Uncertainty</b> Bridge spec is 1ppm or 1 digit. For this measurement 1 digit is larger, this equates to 1.0 mK. This is assumed as a 95% (k=2) coverage.	1.0 mK	B	95.45	2	.50
<b>Bridge Resolution</b> Bridge spec is 1 ppm, the uncertainty limit is ½ of this which equates to 0.5 mK. This is assumed as a 100% confidence rectangular distribution, (k=1.732)	0.5 mK	B	100	1.732	0.29
<b>Standard Resistor Uncertainty</b> Calibration certificate for resistor states an expanded uncertainty of less than 4 ppm with a 95% (k=2) coverage. This equates to 1.0 mK.	1.0 mK	B	95.45	2	0.50
<b>Standard Resistor Drift</b> The drift of the standard resistor has been tracked for multiple years and has performed at better than 1 ppm per year drift, this is assumed as a 95% (k=2) coverage. This equates to 0.25 mK.	0.25 mK	B	95.45	2	0.13
<b>Standard Resistor Thermal Effects</b> The resistor is maintained in a temperature controlled bath that results in 0.3 ppm variation in the resistance, this equates to 0.08 mK. This is assumed as a “U-Shaped” distribution with 100% coverage, k=1.414	0.08 mK	B	100	1.414	0.06
<b>Mathematical Correction Error</b> The error caused by the mathematical addition of .0010 ohms for the nominal .01°C difference between the ice point and TPW would be less than .001 mK which is insignificant in this uncertainty estimation.	0.00 mK	B	95.45	2	0.00
<b>Combined Standard Uncertainty for Equipment Only</b> This does not include Repeatability and Reproducibility component which can be heavily influenced by the UUT.					1.69
<b>Repeatability and Reproducibility</b> R&R for this UUT using this method is 0.54 mK.	0.54 mK	A	63.2	1	0.54
<b>Standard Uncertainty = RSS Total =</b>					<b>1.77</b>
<b>Expanded Uncertainty, 95% (k=2)</b>					<b>3.5 mK</b>

Table A.3B. Uncertainty analysis of a SSPRT by ice bath as 0°C reference source (Method 3).

Source	Source Uncertainty	Type (A/B)	Level of Confidence (%)	k factor	Standard Uncertainty (mK)
<b>Ice Bath Stability and Uniformity</b> Periodic testing of our ice bath shows that it is consistently within 3.0 mK of 0°C. For this analysis stability and uniformity will be considered to be 3.0 mK with a 95% (k=2) coverage.	3.0 mK	B	95.45	2	1.50
<b>Bridge Uncertainty</b> Bridge spec is 1ppm or 1 digit. For this measurement 1 ppm is the same as 1 digit, this equates to 0.25 mK. This is assumed as a 95% (k=2) coverage.	0.25 mK	B	95.45	2	0.13
<b>Bridge Resolution</b> Bridge spec is 1 ppm, the uncertainty limit is ½ of this which equates to 0.13 mK. This is assumed as a 100% confidence rectangular distribution, (k=1.732)	0.13 mK	B	100	1.732	0.08
<b>Standard Resistor Uncertainty</b> Calibration certificate for resistor states an expanded uncertainty of less than 4 ppm with a 95% (k=2) coverage. This equates to 1.0 mK.	1.0 mK	B	95.45	2	0.50
<b>Standard Resistor Drift</b> The drift of the standard resistor has been tracked for multiple years and has performed at better than 1 ppm per year drift, this is assumed as a 95% (k=2) coverage. This equates to 0.25 mK.	0.25 mK	B	95.45	2	0.13
<b>Standard Resistor Thermal Effects</b> The resistor is maintained in a temperature controlled bath that results in 0.3 ppm variation in the resistance, this equates to 0.08 mK. This is assumed as a "U-Shaped" distribution with 100% coverage, k=1.414	0.08 mK	B	100	1.414	0.06
<b>Mathematical Correction Error</b> The error caused by the mathematical addition of .0040 ohms for the nominal .01°C difference between the ice point and TPW would be less than .02 mK. This is assumed as a 95% (k=2) coverage.	0.02 mK	B	95.45	2	0.01
<b>Combined Standard Uncertainty for Equipment Only</b> This does not include Repeatability and Reproducibility component which can be heavily influenced by the UUT.					1.59
<b>Repeatability and Reproducibility</b> R&R for this UUT using this method is 0.39 mK	0.39 mK	A	63.2	1	0.39
<b>Standard Uncertainty = RSS Total =</b>					1.64
<b>Expanded Uncertainty, 95% (k=2)</b>					3.3 mK

Table A.3C. Uncertainty analysis of an IPRT by ice bath as 0°C reference source (Method 3).

Source	Source Uncertainty	Type (A/B)	Level of Confidence (%)	k factor	Standard Uncertainty (mK)
<b>Ice Bath Stability and Uniformity</b> Periodic testing of our ice bath shows that it is consistently within 3.0 mK of 0°C. For this analysis stability and uniformity will be considered to be 3.0 mK with a 95% (k=2) coverage.	3.0 mK	B	95.45	2	1.50
<b>Bridge Uncertainty</b> Bridge spec is 1ppm or 1 digit. For this measurement 1 ppm is the same as 1 digit, this equates to 0.25 mK. This is assumed as a 95% (k=2) coverage.	0.25 mK	B	95.45	2	0.13
<b>Bridge Resolution</b> Bridge spec is 1 ppm, the uncertainty limit is ½ of this which equates to 0.13 mK. This is assumed as a 100% confidence rectangular distribution, (k=1.732)	0.13 mK	B	100	1.732	0.08
<b>Standard Resistor Uncertainty</b> Calibration certificate for resistor states an expanded uncertainty of less than 4 ppm with a 95% (k=2) coverage. This equates to 1.0 mK.	1.0 mK	B	95.45	2	0.50
<b>Standard Resistor Drift</b> The drift of the standard resistor has been tracked for multiple years and has performed at better than 1 ppm per year drift, this is assumed as a 95% (k=2) coverage. This equates to 0.25 mK.	0.25 mK	B	95.45	2	0.13
<b>Standard Resistor Thermal Effects</b> The resistor is maintained in a temperature controlled bath that results in 0.3 ppm variation in the resistance, this equates to 0.08 mK. This is assumed as a "U-Shaped" distribution with 100% coverage, k=1.414	0.08 mK	B	100	1.414	0.06
<b>Mathematical Correction Error</b> The error caused by the mathematical addition of .0039 ohms for the nominal .01°C difference between the ice point and TPW would be less than .02 mK. This is assumed as a 95% (k=2) coverage.	0.02 mK	B	95.45	2	0.01
<b>Combined Standard Uncertainty for Equipment Only</b> This does not include Repeatability and Reproducibility component which can be heavily influenced by the UUT.					1.59
<b>Repeatability and Reproducibility</b> R&R for this UUT using this method is 0.46 mK	0.46 mK	A	63.2	1	0.46
<b>Standard Uncertainty = RSS Total =</b>					<b>1.66</b>
<b>Expanded Uncertainty, 95% (k=2)</b>					<b>3.3 mK</b>

## **References**

1. H. Preston-Thomas, “The International Temperature Scale of 1990 (ITS-90)”, *Metrologia* 27, 3-10, 1990, p. 6.
2. ASTM International, Standard E 563-02 “Standard Practice for Preparation and Use of an Ice-Point Bath as a Reference Temperature”, *Annual Book of ASTM Standards Vol. 14.03*, 2007, pp. 343-346.
3. ASTM International, Standard E 1750-02, “Standard Guide for Use of Water Triple Point Cells”, *Annual Book of ASTM Standards Vol. 14.03*, 2007, pp. 526-532.